

CUTE solutions for two-point correlation functions from large cosmological datasets

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In the advent of new large galaxy surveys, which will produce enormous datasets with hundreds of millions of objects, new computational techniques are necessary in order to extract from them any two-point statistic, the computational time of which grows with the square of the number of objects to be correlated. Fortunately technology now provides multiple means to massively parallelize this problem. Here we present a free-source code specifically designed for this kind of calculations. Two implementations are provided: one for execution on shared-memory machines using OpenMP and one that runs on graphical processing units (GPUs) using CUDA. The code is available at <http://members.ift.uam-csic.es/dmonge/CUTE.html>.

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I. INTRODUCTION

The last decades have seen an increasing interest in galaxy surveys as a means of studying the late-time evolution of the Universe. Forthcoming galaxy surveys, such as DES [1], BigBOSS [2] or Euclid [3], will map large regions of the sky ($\mathcal{O}(10^{3-4})$ sq-deg) to redshifts $z > 1$ yielding catalogs containing hundreds of millions of objects.

The spatial distribution of these objects on different scales contains invaluable information that could help clarify many open problems in cosmology and astrophysics, such as the nature of dark matter and dark energy or the presence of primordial non-Gaussianities in the density field. One of the simplest observables that can be estimated to quantify the clustering of matter on different scales is the two-point correlation function (2PCF hereon, see section II). Its estimation is based on counting pairs of objects separated by a given distance measure, and therefore its computational time grows with the square of the number of objects in the catalog. Hence, when $\mathcal{O}(10^{14-16})$ pairs must be considered, a simplistic serial approach is too slow for the full-scale problem, and, besides using some simplifying approximation, the only viable solution becomes parallelising the calculation. In this sense modern graphical processing units (GPUs) provide the means to perform many operations in parallel on a large number (hundreds) of cores with a moderate clock frequency for a comparatively cheap price. Another approach is using a relatively smaller number of high-frequency CPU cores both in shared or distributed memory machines.

Here we present a CUTE (Correlation Utilities and Two-point Estimation), a free open-source code that estimates different kinds of two-point correlations from discrete cosmological catalogs using different speed-up techniques.

II. THE TWO-POINT CORRELATION FUNCTION(S)

The three-dimensional 2PCF $\xi(\mathbf{r})$ of a set of discrete points in \mathbb{R}^3 represents the excess probability of finding two of them inside two small volumes dV_1 and dV_2 separated by \mathbf{r} [4]:

$$\langle dP \rangle = \bar{n}[1 + \xi(\mathbf{r})] dV_1 dV_2. \quad (1)$$

When this point distribution comes from a Poisson process based on an underlying random density field $\delta(\mathbf{x})$, the field's 2PCF $\xi_\delta(\mathbf{r}) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r}) \rangle$ is directly related to that of the point distribution [5]. Note that even though, in principle, the two-point correlation should depend on the positions of both points, \mathbf{r}_1 and \mathbf{r}_2 , for homogeneous fields the only dependence is on the separation between them $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$.

A. Types of correlation functions

- **The 3-D correlation function $\xi(r, \mu)$ and $\xi(\sigma, \pi)$.** Different observational effects such as redshift space distortions or errors in the observed redshifts transform what would otherwise be an isotropic 2PCF into a function that behaves differently along the line of sight and in the transverse direction. Two coordinate systems are widely used in the literature: the $\sigma - \pi$ and $r - \mu$ schemes (see fig. 1), the relation between both being

$$\pi = r \mu, \quad \sigma = \sqrt{r^2 - \pi^2}. \quad (2)$$

The $r - \mu$ scheme has the advantage that the usual multipole expansion is directly written in terms of these variables:

$$\xi(r, \mu) = \sum_l \xi_l(r) P_l(\mu), \quad (3)$$

where P_l are the Legendre polynomials. Note that at the linear level and in the plane-parallel approximation (i.e. the Kaiser formula [6]) only the first three even multipoles ($l = 0, 2, 4$) contribute.

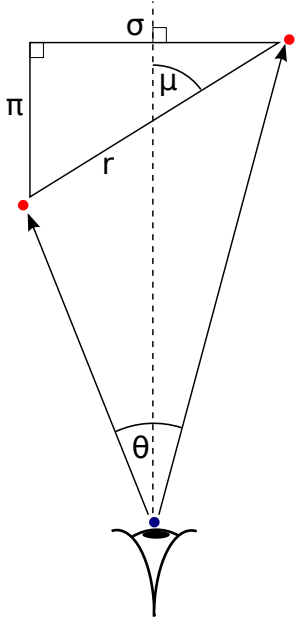


FIG. 1: Definition of the different coordinate conventions used.

- **The monopole** $\xi_0(r)$. The first element ($l = 0$) in the expansion above is the angle-averaged correlation function or “monopole”:

$$\xi_0(r) = \frac{1}{2} \int_{-1}^1 d\mu \xi(r, \mu) \quad (4)$$

This is the only non-zero contribution in the absence of redshift distortions.

- **The radial correlation function** $\xi_r(\bar{z}, \Delta z)$. Correlating only pairs of galaxies aligned with the line of sight one computes the so-called radial correlation function, which can be made to depend locally only on the redshift difference Δz between each pair of galaxies. It is related to the three-dimensional 2PCF as

$$\xi_r(\bar{z}, \Delta z) = \xi(\pi(\bar{z}, \Delta z), \sigma = 0), \quad (5)$$

where

$$\pi(\bar{z}, \Delta z) = \chi(\bar{z} + \Delta z/2) - \chi(\bar{z} - \Delta z/2) \simeq \frac{c \Delta z}{H(\bar{z})}, \quad (6)$$

where $\chi(z)$ is the radial comoving distance to redshift z .

- **The angular correlation function** $w(\theta)$. The angular correlation function is the 2PCF of the density contrast field projected on the sphere

$$w(\theta) \equiv \langle \delta_s(\hat{\mathbf{n}}_1) \delta_s(\hat{\mathbf{n}}_2) \rangle, \quad \cos \theta \equiv \hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2, \quad (7)$$

$$\delta_s(\hat{\mathbf{n}}) \equiv \int dz \phi(z) \delta(r(z) \hat{\mathbf{n}}), \quad (8)$$

where $\phi(z)$ is the redshift selection function. The angular correlation function is related to $\xi(r, \mu)$ by

$$w(\theta) = \int dz_1 \phi(z_1) \int dz_2 \phi(z_2) \xi(r(z_1, z_2, \theta), \mu(z_1, z_2, \theta)) \quad (9)$$

$$r(z_1, z_2, \theta) = \sqrt{\chi^2(z_1) + \chi^2(z_2) - 2\chi(z_1)\chi(z_2)\cos\theta},$$

$$\mu(z_1, z_2, \theta) = \frac{|\chi^2(z_1) - \chi^2(z_2)|}{\sqrt{(\chi^2(z_1) + \chi^2(z_2))^2 - 4\chi^2(z_1)\chi^2(z_2)\cos^2(\theta)}}$$

B. Estimating the 2PCF from discrete data

When calculating the correlation function from a point distribution several observational difficulties arise related to the survey selection function (or more generally, to the spatial distribution of the data): different parts of the sky may have been mapped to different depths and the radial (redshift) distribution is never uniform. The most usual technique to deal with these issues is to compare the data catalog with catalogs made of randomly distributed objects. Thus, since the correlation function is the excess probability, with respect to a random distribution, of finding two objects within a given distance, it can be naively estimated as

$$\xi_N = \frac{N_r(N_r - 1)}{N_d(N_d - 1)} \frac{DD}{RR} - 1 \quad (10)$$

where N_d and N_r are the number of points in the data and random catalogs respectively and DD and RR are histograms containing the counts of pairs of objects found separated by a given distance in each catalog. This naive (or natural) estimator is in fact biased and several other ones have been proposed in the literature, making use not only of DD and RR but also of the cross-correlation of random and data DR . The most widely accepted estimator is the one proposed by Landy & Szalay [7]:

$$\xi_{LS} = \frac{\frac{N_r(N_r-1)}{N_d(N_d-1)} DD - \frac{N_r-1}{N_d} DR + RR}{RR}, \quad (11)$$

see [8] for a thorough comparison of different estimators.

The most delicate part of the estimation is in fact being able to generate the random catalogs correctly: the background spatial distribution, both in angles and redshift (i.e. the one-point function), of random objects must be exactly the same as in the data. Only then are mask-related effects accurately corrected. Also, in order to minimize Poisson errors in DR and RR , random catalogs should be generated with more particles than the data.

III. CUTE

CUTE (Correlation Utilities and Two-point Estimation) is a free and open-source code for cosmological 2PCF es-

timization. CUTE is written in C and, in the current public version, comes with two implementations: one parallelized for shared-memory machines using OpenMP and one (CU.CUTE) that performs the correlations in a GPU using NVidia's CUDA architecture. CUTE calculates 4 different correlation functions (3-D, monopole, angular and radial) with different binning schemes and speed-up techniques. Here we will explain the parallelization strategies followed by CUTE and some details specific to each type of 2PCF. We refer the reader to the README file accompanying the latest public version of CUTE for the operational options and compilation instructions of the code. In this section we assume some basic knowledge of parallel computing with OpenMP and CUDA by the reader.

It must be noted that there exist two other codes [9, 10], recently made public, designed to compute angular correlation functions with GPUs. As in the case of CUTE the speed-up factor (about 10^2) gained by these codes through the use of graphical devices for parallelization clearly makes it worth the effort of adapting CPU algorithms to run on GPUs.

A. Serial approach

Once the random catalog has been produced, DD , DR and RR are computed by autocorrelating or crosscorrelating each pair of catalogs. In a serial code this algorithm is extremely simple, involving one loop over each catalog and performing 3 operations in each iteration: calculating the distance between each pair of objects, determining the bin corresponding to that distance and increasing the histogram count on that bin. The corresponding C-code would be:

```

1 int histogram[nbins];
2 for(i=0;i<np1;i++) {
3     for(j=0;j<np2;j++) {
4         //Calculate distance between two objects
5         double dist=get_dist(x1[i],y1[i],z1[i],
6                               x2[j],y2[j],z2[j]);
7         //Calculate bin number
8         int ibin=bin_dist(dist);
9         //Increase histogram count
10        histogram[ibin]++;
11    }
12 }
```

As we said before, these two nested loops make this an N^2 problem (to be precise, an $n1*n2$ problem), whose computational time will grow very fast as we increase the size of the catalogs. At this point parallelization or some kind of fast approximate method, such as the use of k -Trees or pixels (see section III C 2) is desirable if not compulsory. The approach taken by CUTE is to parallelize one of the two loops, which introduces complications in this simple algorithm.

B. Parallelization with CUDA and OpenMP

1. Multicore shared-memory machines and OpenMP

OpenMP [19] is an API that gives support for parallel programming in shared-memory platforms. Once a parallel execution block is opened, the programmer can define private (one independent copy per core) or shared (common) variables and easily divide `for` loops between all available cores. For a thorough review of the different features of OpenMP see [11]. The serial code above takes the following form when parallelized with OpenMP:

```

1 int histogram[nbins];
2 int histo_thread[nbins];
3 #pragma omp parallel default(none) \
4   private(hthread) shared(...) {
5     //Initialize private histograms
6     for(i=0;i<nbins;i++)
7         histo_thread[nbins]=0;
8 #pragma omp for //Parallelize loop
9     for(i=0;i<np1;i++) {
10        for(j=0;j<np2;j++) {
11            //Calculate distance between two objects
12            double dist=get_dist(x1[i],y1[i],z1[i],
13                                 x2[j],y2[j],z2[j]);
14            //Calculate bin number
15            int ibin=bin_dist(dist);
16            //Increase histogram count
17            histo_thread[ibin]++;
18        }
19    }
20 #pragma omp critical {
21     //Add private histograms
22     for(i=0;i<nbins;i++)
23         histogram[i]+=histo_thread[i]
24 }
25 }
```

The strategy in this case is to declare one private histogram per execution thread that will store that thread's pair counts. The first loop is then divided between all available threads and finally all partial histograms are added, avoiding read/write collisions, into the final shared one. As can be seen, parallelization with OpenMP is effortless, only requiring a few extra lines of code.

2. Graphics cards and CUDA

A GPU (Graphical Processing Unit) is a specialized piece of hardware designed for fast massively parallel manipulation of memory addresses. As their name suggests, GPUs are mainly intended for image building and processing, however their highly parallel structure makes them ideal for intensive numerical computation, providing a relatively cheap flop/s (floating-point operations per second). Hence in the last years GPUs have found their way into different branches of scientific research, computational cosmology being one of them [12, 13] Initially the main difficulty when trying to use GPUs for scientific computing was the programming of the numerical algorithm, since using the standard APIs meant that

data had to be disguised as pixel colors and some mathematical operations had to be encoded as graphics rendering. However lately a few programming models for GPUs have seen the light of day [20] [21] that make general purpose computing on GPUs (GPGPU) a lot easier. Of these we have chosen Nvidia's CUDA [14] for its syntactic simplicity.

Two main complications arise when one tries to adapt a code to execute on a GPU. First, in a massively parallel environment one must take great care to avoid race conditions due to simultaneous memory read/write processes by different threads. Second, unlike in a multi-CPU machine, the amount of memory “per thread” available in a GPU is very limited, presently of the order of a few GB for hundreds of processors. Besides this there are other more subtle concerns, such as intra-warp communication or the presence of different types of cached and uncached memory in the GPU, the correct use of which may enhance dramatically the code's performance. In summary, correctly parallelising a code with CUDA is not as straightforward as it is with OpenMP, and in some cases it may not be worth the effort. For an introduction to CUDA and its many features see [15].

Implementing the serial algorithm above in CUDA would involve executing the following `--device--` function in every thread in parallel:

```

1  __shared__ int histo_thread[nbins];
2  int stride=blockDim.x*gridDim.x;
3  //Initialize shared histogram
4  histo_thread[threadIdx.x]=0;
5  __syncthreads();
6  //Correlate
7  for(i=0;i<np1;i++) {
8      int j=threadIdx.x+blockIdx.x*blockDim.x;
9      while(j<np2) {
10         //Calculate distance between two objects
11         double dist=get_dist(x1[i],y1[i],z1[i],
12                             x2[j],y2[j],z2[j]);
13         //Calculate bin number
14         int ibin=bin_dist(dist);
15         //Increase histogram count
16         atomicAdd(&(histo_thread[ibin]),1);
17         //Increase second index by stride
18         j+=stride;
19     }
20 }
21 //Add block histograms
22 __syncthreads();
23 atomicAdd(&(histogram[threadIdx.x]),
24          histo_thread[threadIdx.x]);

```

As before, we have divided one of the loops (this time the second one) among all the execution threads. The first difference with respect to OpenMP that we can see immediately is that, due to the limited amount of memory of the GPU, we can only declare one partial histogram per block, and not per thread. To do this we declare it as a variable in shared memory, which also has the advantage of having a lower latency than global memory. This introduces a new complication, since now all threads in a block will try to add their pair counts to the same histogram. This has to be done avoiding race conditions by using the CUDA `atomicAdd()` function.

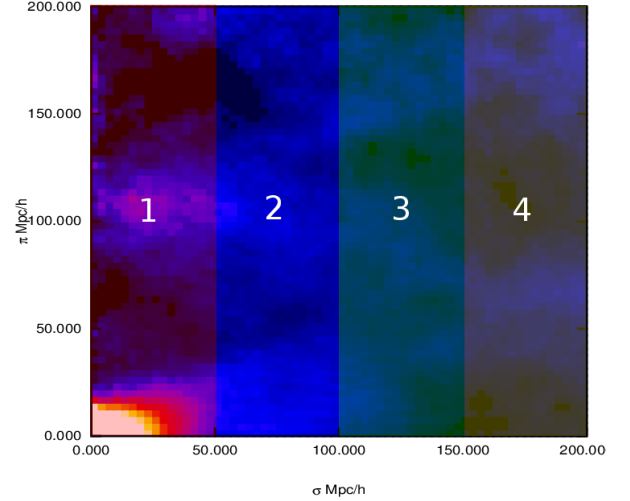


FIG. 2: Illustration of the method used to calculate the 3-D correlation function in CUDA. Since the whole 2-dimensional 128×128 bin histogram cannot be fit inside the device's shared memory, it is split into smaller ones, which are filled separately. Although the catalogs have to be correlated more than once, the bottleneck caused by atomic operations is largely mitigated by the higher number of histogram bins.

This is in fact the bottleneck of any algorithm involving histograms in CUDA (especially if the distribution under study is very degenerate), since many threads may have to remain idle while waiting for other threads to update their histogram entries. The best way to palliate this problem is to use at most as many threads per block as histogram bins (in fact, note that the algorithm above will only work when using as many threads as histogram bins, however it can be easily extended to more general cases). Finally all the partial block histograms are summed up into the global histogram in an ordered manner using again `atomicAdd()`. The fact that CUTE uses CUDA atomic functions such as `atomicAdd()` means that it will only run on GPUs that support atomic operations (namely compute capability 2.0 or higher). There exist general algorithms for histograms that work on any CUDA-enabled device [16, 17], however no performance improvement was observed with respect to using `atomicAdd()`. Furthermore, the method above reduces the use of shared memory for histograms to a minimum, allowing its use for other purposes.

C. Specifics of the 2PCFs

In the previous section we have described the general strategy followed to parallelize the calculation of any 2PCF with OpenMP and CUDA. However each of the different 2PCFs detailed in section II requires a different treatment of the data and maybe allows for different, more optimal, approaches. We give the details specific to each of these types here.

1. Radial correlation function

As was said in section II the radial 2PCF is calculated by correlating pairs of aligned objects and binning them according to their relative redshift difference Δz . The approach taken by CUTE to define aligned pairs is to divide the sky into small angular pixels and to correlate only pairs of objects belonging to the same pixel. For reasonable pixel sizes (< 1 sq-deg) the number of objects to correlate per pixel is relatively small ($\mathcal{O}(10^2-3)$ for expected catalog sizes). Hence, since the computational time in this case is not an issue, there is no need for massive parallelization, and radial correlation functions are only supported by CUTE in its OpenMP version.

2. Angular correlation function

For the calculation of angular 2PCFs CUTE projects all objects in the catalog into the unit sphere and correlates pairs of objects according to their angular separation θ (see figure 1), which is used as a distance measure. Two complementary speed-up techniques can be used by CUTE in this case: if one is not interested in extremely small angular scales one can create a pixel map from the catalog and then correlate the pixels (weighting each of them by the number of objects that fall inside it). This may effectively reduce the number of objects that must be correlated by an order of magnitude and therefore reduce the computational time by a factor of 100. Also, in the calculation of the angular separation, the arc-cosine of the scalar product of two position vectors must be estimated. Calculating the arc-cosine is a very time-consuming operation, and, if one is not interested in very large angular scales, the following approximation can be used,

$$\arccos(1-x) \sim \sqrt{2x + \frac{1}{3}x^2 + \frac{4}{45}x^3}$$

which is precise to 1 part in 10^{-4} for angles below 40° and reduces the computational time by a factor ~ 2 . Both these time-saving techniques can be switched on or off in CUTE by the user.

3. 3-D correlation function

The main difference in the calculation of the 3-D correlation function with respect to the other 2PCFs is that pairs are binned in 2-dimensional histograms, according to their (r, μ) or (π, σ) separations. This does not introduce any relevant changes in the OpenMP implementation, as long as the amount of shared memory is large enough to accomodate one private 2-D histogram per thread, however it does matter when adapting the code to CUDA. The reason is that currently the amount of shared memory per block in GPUs is limited to 48 kB,

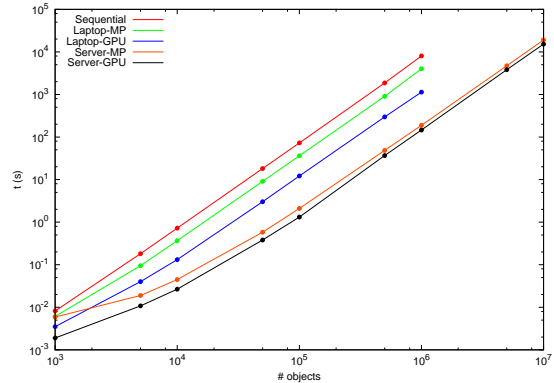


FIG. 3: Computational times employed by different devices to compute the monopole 2PCF of catalogs of different sizes. A speed-up factor of $\mathcal{O}(100)$ can be gained by using a high-end GPU with respect to a sequential approach on a high-end CPU. Even with a regular gaming GPU the increase in speed is substantial ($\mathcal{O}(10)$). The different devices are described in table I.

which is too little to allocate, for example, a 128×128 array of long integers. The solution to this problem chosen for CUTE is explained in fig. 2: the catalogs are correlated several times, each time binning only pairs whose separation in one of the two coordinates is within a given range, until the whole histogram is filled. Thus we can declare smaller 2-D histograms in shared memory, and even though the catalogs must be correlated several times, the histogram-filling bottleneck mentioned in section III B 2 is largely alleviated by the higher number of histogram entries, thus conserving a reasonable computational time (see section IV).

IV. PERFORMANCE

We have tested CUTE's performance in terms of computational time by running it on platforms with different capabilities, listed in table I. The serial version was tested by running CUTE on a single CPU core. We also tested the OpenMP version on a dual-core laptop and on a large shared-memory machine with 80 cores. The CUDA version has been tried on a regular graphics card in a laptop and on a high-end GPU. For this test the monopole 2PCF was calculated for catalogs of different sizes in the range $10^3 - 10^7$. The computational times for one single correlation (i.e. just calculating, for example, DR) in the 5 different platforms are plotted in figure 3. As expected, using GPUs or parallelising the computation on several CPU cores improves the code's speed by a factor 10 - 100, even using a regular video-game graphics card. The elapsed times were measured using OpenMP and CUDA timing functions, since these

	Name	Description	#cores
CPUs	Sequential	Intel Core i7-2620M	1 core (\equiv 1 thread)
	Laptop-MP	Intel Core i7-2620M	2 cores (\equiv 4 threads)
	Server-MP	Intel MP NEHALEM-EX ($\times 8$)	80 cores (\equiv 160 threads)
GPUs	Laptop-GPU	NVIDIA NVS 4200M	48 CUDA cores
	Server-GPU	NVIDIA TESLA C2070 FERMI	448 CUDA cores

TABLE I: Different devices in which CUTE has been tested: a single CPU core, a dual core, a multi-core shared-memory machine (160 threads), an ordinary graphics card and a high-end GPU.

Platform	$T(\xi(r))$	$T(\xi_{\log}(r))$	$T(w(\theta))$	$T(w_{\text{pix}}(\theta))$	$\xi(\sigma, \pi)$
Sequential	877	5230	1374	21	2238
Laptop-MP	389	2676	628	5.3	1064
Laptop-GPU	113	185	283	6.2	297
Server-MP	25	52	32	0.51	50
Server-GPU	13	20	22	0.46	27

TABLE II: Computational times elapsed, for each of the 5 platforms listed in table I, during the calculation of 5 different 2PCFs: monopole ($\xi(r)$), monopole with logarithmic binning ($\xi_{\log}(r)$), angular ($w(\theta)$), angular with pixels of resolution $\Delta\Omega \equiv 5 \times 10^{-3}$ sq-deg ($w_{\text{pix}}(\theta)$) and 3-D ($\xi(\sigma, \pi)$). Times are in seconds and correspond to the calculation of the *DR* histogram (the full calculation of the 2PCF should take 2-3 times longer).

give the most accurate estimate of the time spent doing the actual correlation.

For completeness we have also listed in table II the computational times taken by the 5 different devices to calculate different correlation functions. The dataset used for this exercise is a subset of one of the mock catalogs provided by the MICE project [22] [18], with $0^\circ < \text{dec} < 18^\circ$, $0^\circ < \text{R.A.} < 18^\circ$, $0.5 < z < 0.6$, containing $\sim 3 \times 10^5$ particles. The 5 different correlation functions are:

- Monopole correlation function: linear binning for $r < 100 \text{ Mpc}/h$ and 256 bins.
- Monopole correlation function: logarithmic binning for $r < 100 \text{ Mpc}/h$ using 256 bins and 50 bins per decade.
- Angular correlation function: linear binning for $\theta < 30^\circ$ and 256 bins. Calculated by brute-force.
- Angular correlation function: linear binning for $\theta < 30^\circ$ and 256 bins. Calculated using pixels with resolution $\Delta\Omega \equiv 5 \times 10^{-3}$ sq-deg.
- 3-D correlation function: binning in (π, σ) on a 64×64 -bin histogram.

Examples of the output produced by CUTE for the different kinds of correlation functions can be seen in figures

4 and 5.

V. SUMMARY

We have presented CUTE, a parallel code for computing two-point correlation functions from cosmological catalogs. CUTE has been optimized to run on shared-memory machines as well as graphical processing units. It can estimate the 3-D, monopole, radial and angular correlation functions from a set of data using different speed-up techniques and binning schemes. We have shown that great benefits in terms of computational speed can be gained by parallelising the algorithm on GPUs.

The code is publicly available through our website <http://members.ift.uam-csic.es/dmonge/CUTE.html>. CUTE is released under the GNU Public License (GPL).

Acknowledgments

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[1] The Dark Energy Survey Collaboration, 2005, arXiv:astro-ph/0510346

[2] Schlegel D. et al., 2011, arXiv:1106.1706 [astro-ph.IM]

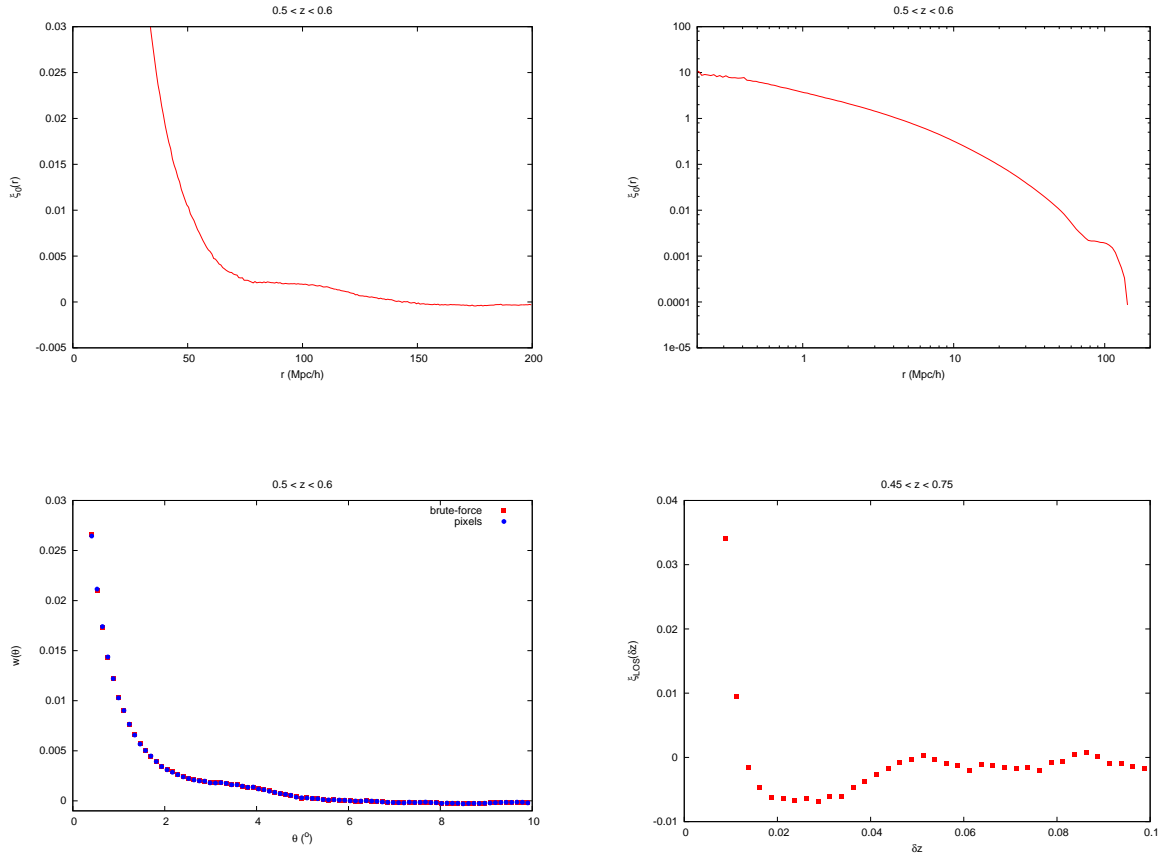


FIG. 4: Types of correlation functions calculated by CUTE. Monopole with linear (top left) and logarithmic binning (top right), angular (bottom left) and radial 2PCFs (bottom right). They were extracted from different redshift slices (specified at the top of each plot) of the MICE mock catalog. The angular correlation function was calculated both by brute-force and using pixels. Although the latter method is a lot faster (see table II) both results agree very well on the scales shown.

- [3] Laureijs R. et al., 2011, arXiv:1110.3193
- [4] Peebles P. J. E., 1980, The large-scale structure of the universe, Princeton University Press
- [5] Martínez V., Saar E., 2001, Statistics of the galaxy distribution, Chapman & Hall/CRC
- [6] Kaiser N., 1987, MNRAS, 227, 1
- [7] Landy S. D., Szalay A. S., 1993, ApJ, 412, 64
- [8] Kerscher M., Szapudi I., Szalay A. S., 2000, ApJ, 535, L13
- [9] Ponce R., Cardenas-Montes M., Rodríguez-Vázquez J., Sánchez E., Sevilla I., 2011, Proc. XXI ADASS Conf., Paris. arXiv:1204.6630
- [10] Bard D., Bellis M., Allen M. T., Yepremyan H., Kratochvil J. M., arXiv:1208.3658 [astro-ph.IM].
- [11] Chapman B., Jost G., Van Der Pas R., 2008, Using OpenMP - Portable Shared Memory Parallel Programming, The MIT Press
- [12] Bedorf J., Gaburov E., Zwart S. P., 2012, arXiv:1204.2280
- [13] Nakasato N., 2011, arXiv:1112.4539
- [14] Nickols et al., 2008, Queue 6, 2, 40-53
- [15] Sanders J., Kandrot E., 2011, CUDA by example - An introduction to General-Purpose GPU Programming, Addison-Wesley
- [16] Podlozhnyuk V., 2007, NVIDIA, Tech. Rep.
- [17] Shams R., Kennedy R. A., 2007, Proc. ISPCS, 418-422
- [18] Fosalba P. Gaztañaga E., Castander F., Manera M. 2008, MNRAS, 391, 435
- [19] <http://www.openmp.org>
- [20] http://www.nvidia.com/object/cuda\protect_home.html
- [21] <http://www.khronos.org/opencv/>
- [22] <http://maia.ice.cat/mice/>

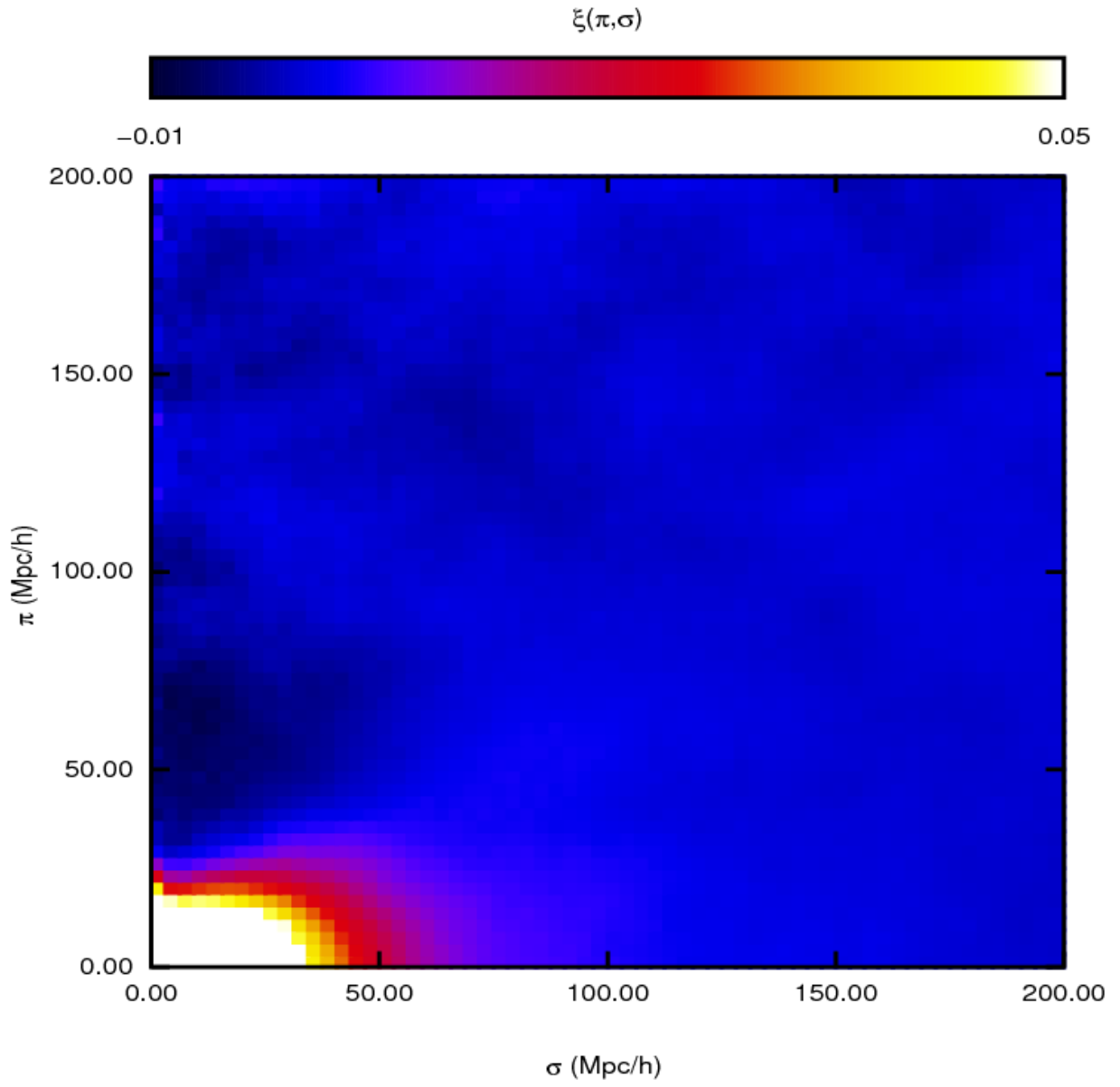


FIG. 5: 3-D correlation function calculated from the MICE mock catalog.